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CALCULUS.

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261. Proposed by S. A. COREY, Hiteman, Iowa.

Prove that $\sum_{x=1}^{x=\infty}\frac{1}{a+2bx^2+cx^4}=\frac{\pi}{\sqrt{[8ac(\sqrt{ac}+b)]}}-\frac{1}{2a}$, where $ac>b^2$.

Solution by V. M. SPUNAR, M. and E. E., East Pittsburg, Pa.

$$\sum_{x=1}^{x=\infty}\frac{1}{a+2bx^2+cx^4}=\int_1^\infty\frac{dx}{a+2bx^2+cx^4}=A.$$

As $ac>b^2$, $a+2bx^2+cx^4=(\sqrt{a}+2kx+x^2\sqrt{c})(\sqrt{a}-2kx+x^2\sqrt{c})$, where $k=\sqrt{\frac{\sqrt{(ac)}-b}{2}}$.

$$\therefore A=\frac{1}{4k\sqrt{a}}\int_1^\infty\frac{(x+2k)dx}{\sqrt{a}+2kx+x^2\sqrt{c}}-\frac{1}{4k\sqrt{a}}\int_1^\infty\frac{(x-2k)dx}{\sqrt{a}-2kx+x^2\sqrt{c}}$$

$$=\left[\frac{1}{8k\sqrt{a}}\log\frac{\sqrt{a}+2kx+x^2\sqrt{c}}{\sqrt{a}-2kx+x^2\sqrt{c}}+\frac{1}{4\sqrt{[a(b+k^2)]}}\tan^{-1}\frac{2x\sqrt{(b+k^2)}}{\sqrt{a}-x^2\sqrt{c}}\right]_1^\infty$$

$$=\tfrac{1}{2}\left[\frac{\pi}{\sqrt{ah}}-\frac{1}{a}\right],\text{ where } h=2\sqrt{(ac)}+b,=\frac{\pi}{\sqrt{[8ac(\sqrt{ac}+b)]}}-\frac{1}{2a}.$$

Also solved by G. B. M. Zerr.

262. Proposed by H. SCHAFFER, Fayetteville, Ark.

Prove that the circle is the only plane curve of constant curvature.

Solution by C. N. SCHMALL, New York City.

The expression for the curvature of a plane curve, $F(x, y)=0$, is

$$\frac{1}{R}=\frac{\dfrac{d^2y}{dx^2}}{\left[1+\left(\dfrac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}=c,\text{ say...(1).}$$

Put $\frac{dy}{dx}=z$. $\because\frac{d^2y}{dx^2}=\frac{dz}{dx}$, and (1) becomes $\frac{dz/dx}{(1+z^2)^{\frac{3}{2}}}=c$, whence

$dx=\frac{dz}{c(1+z^2)^{\frac{3}{2}}}$, and, therefore, $x=\frac{1}{c}\int\frac{dz}{(1+z^2)^{\frac{3}{2}}}=\frac{1}{c}\cdot\frac{z}{\sqrt{(1+z^2)}}$.

$\therefore c^2x^2=\frac{z^2}{1+z^2}$. $\therefore z^2=\frac{c^2x^2}{1-c^2x^2}$, $z=\frac{cx}{\sqrt{(1-c^2x^2)}}$; *i. e.*, $\frac{dy}{dx}=\frac{cx}{\sqrt{(1-c^2x^2)}}$.

$\therefore dy=\frac{cxdx}{\sqrt{(1-c^2x^2)}}$, and $y=c\int\frac{xdx}{\sqrt{(1-c^2x^2)}}=c.\left(-\frac{1}{c^2}\sqrt{(1-c^2x^2)}\right)$

$=-\frac{1}{c}\sqrt{(1-c^2x^2)}$. Squaring, $y^2=\frac{1}{c^2}(1-c^2x^2)$, or $c^2(x^2+y^2)=1$, the equation of a circle.

Also solved by G. B. M. Zerr, and V. M. Spunar.

ERRATUM. In problem 264, Calculus, the proposer evidently meant

$$\left(\frac{d^2\phi}{d\psi^2}\right)^2 \text{ instead of } \left(\frac{d^2\phi}{d^2\psi}\right)^2.$$

MECHANICS.

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215. Proposed by R. D. CARMICHAEL, Anniston, Ala.

Determine the curve in a vertical plane along a chord of which a particle will slide under the force of gravity and the retardation of friction so that it will traverse the whole length of the chord in a time t which is independent of its direction as long as the upper end of the chord remains fixed. Discuss the result.

Solution by J. SCHEFFER, A. M., Kee Mar College, Hagerstown, Md.

Take the fixed end of the chord for the origin of the axes, that of x being horizontal, and that of y vertical. Let s denote the length of any chord drawn from the fixed point, and denote by θ the angle it makes with the horizon. Then, denoting the coefficient of friction by μ, we have $s=g(\sin\theta-\mu\cos\theta)\frac{t^2}{2}$, t being the time. If now, t is to be independent of θ, $\frac{s}{\sin\theta-\mu\cos\theta}$ must be a constant, say$=a$.

$\therefore \dfrac{\sqrt{(x^2+y^2)}}{\dfrac{y}{\sqrt{(x^2+y^2)}}-\dfrac{\mu x}{\sqrt{(x^2+y^2)}}}=a$, or, $x^2+y^2=a(y-\mu x)$, a circle, the coordinates of the center of which are $-\frac{1}{2}a\mu$ and $\frac{1}{2}a$, and radius $=\frac{a}{2}\sqrt{(1+\mu^2)}$.

Also solved by G. B. M. Zerr.

215. Proposed by HENRY WRITT, Genoa Junction, Wisconsin.

Suppose two centers of attractive forces A and B having a ratio 1 : 330,000, and influence reducing as the second power of the distance, *i. e.*,